Design of Experiments Basic Concepts

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Experimental Objectives

- **“Is this better than that?”**
- **“What matters?”**
- **“Which combination is best?”**

The First Thing You Need

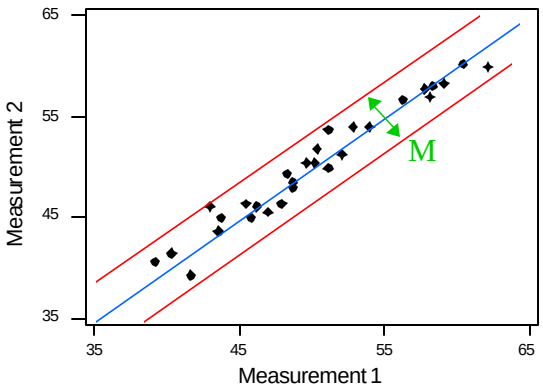
A way to measure the results...

Response

- **Quantitative**
- **Precise**
- **Meaningful**

Isoplot -

Want Discrimination, D > 6



$D \approx \text{Range}(X) \div M$

Common Sense Approach

“Is this better than that?”

The Test Run

I tried it,
but I don’t know what happened.

Minimum Characteristics of an Experiment

- **Comparison -**
  draw valid conclusions
- **Replication -**
  estimate experimental error
- **Randomization -**
  avoid systematic errors

Shainin's Six Pack

- 3 trials of **Current**,
  3 trials of **Proposed**
- Arrange results in order

Worst Best

- Only adopt Proposed if order is as shown above (5% significance)

Common Sense Approach

"What matters?"

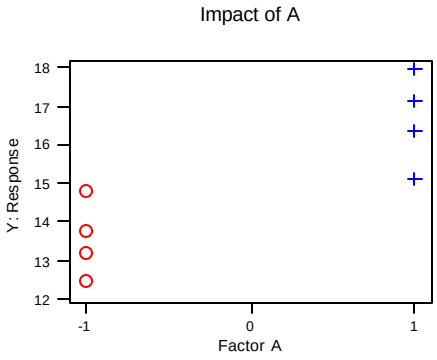
"Which combination is best?"

One Factor-At-A-Time Method

I don't want to get confused.

Factorial Approach -
Orthogonal Contrasts

A	B	Y
-1	-1	y_1
-1	-1	y_2
-1	+1	y_3
-1	+1	y_4
+1	-1	y_5
+1	-1	y_6
+1	+1	y_7
+1	+1	y_8



Factorial Approach -
Orthogonal Contrasts

A	B	Y
-1	-1	y_1
-1	-1	y_2
-1	+1	y_3
-1	+1	y_4
+1	-1	y_5
+1	-1	y_6
+1	+1	y_7
+1	+1	y_8

Effect of A

$= (y_5+y_6+y_7+y_8-y_1-y_2-y_3-y_4) \div 4$

t = Effect ÷ Std. Error

Std. Error = Error Est. ÷ $\sqrt{2}$

Effect significant if

$|t| > t^*_{\alpha/2,(\#cells)(\#reps-1)}$

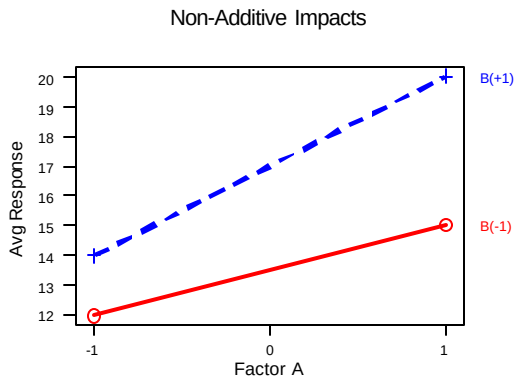
Factorial Approach -
Orthogonal Contrasts

	B(-1)	B(+1)
A(-1)	y_1 y_2 s^2_{--}	y_3 y_4 s^2_{-+}
A(+1)	y_5 y_6 s^2_{+-}	y_7 y_8 s^2_{++}

Error Estimate

$$= \{(s^2_{--} + s^2_{-+} + s^2_{+-} + s^2_{++}) \div 4\}^{1/2}$$

Interactions



Interaction

Orthogonal Contrast

A	B	AB	Y
-1	-1	+1	y_1
-1	-1	+1	y_2
-1	+1	-1	y_3
-1	+1	-1	y_4
+1	-1	-1	y_5
+1	-1	-1	y_6
+1	+1	+1	y_7
+1	+1	+1	y_8

Effect of AB

$= (y_1+y_2+y_7+y_8-y_3-y_4-y_5-y_6)\div 4$

t = Effect ÷ Std. Error

Std. Error = Error Est. ÷ $\sqrt{2}$

Effect significant if

$|t| > t^*_{\alpha/2,(\#cells)(\#reps-1)}$

Efficient Experiments

Get the Required Information with the Least Expenditure of Resources

Fractional Factorial Approach - Confounding

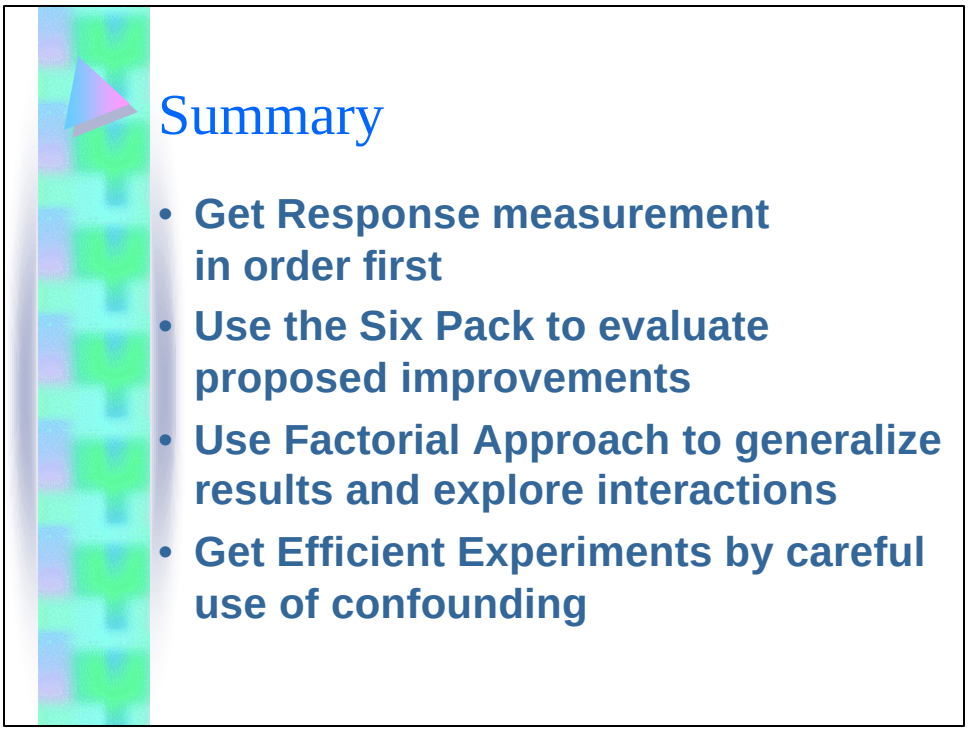
A	B	C	D=ABC	Y	A	B	C	D=ABC	Y
-1	-1	-1	-1	y_1	-1	-1	+1	+1	y_9
-1	-1	-1	-1	y_2	-1	-1	+1	+1	y_{10}
-1	+1	-1	+1	y_3	-1	+1	+1	-1	y_{11}
-1	+1	-1	+1	y_4	-1	+1	+1	-1	y_{12}
+1	-1	-1	+1	y_5	+1	-1	+1	-1	y_{13}
+1	-1	-1	+1	y_6	+1	-1	+1	-1	y_{14}
+1	+1	-1	-1	y_7	+1	+1	+1	+1	y_{15}
+1	+1	-1	-1	y_8	+1	+1	+1	+1	y_{16}

Fractional Factorial Approach - Confounding

I = ABCD (defining contrast)

Alias structure -

A = BCD	AB = CD
B = ACD	AC = BD
C = ABD	BC = AD
D = ABC	


Summary
• Get Response measurement in order first
• Use the Six Pack to evaluate proposed improvements
• Use Factorial Approach to generalize results and explore interactions
• Get Efficient Experiments by careful use of confounding